

Shortest Path & Application

Group 4

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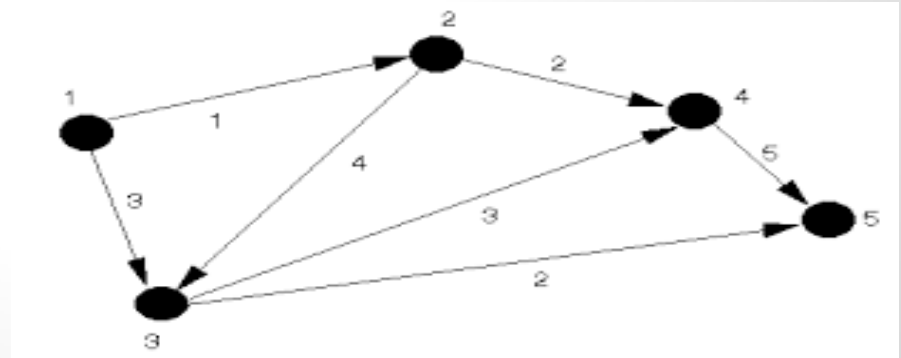
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Overview

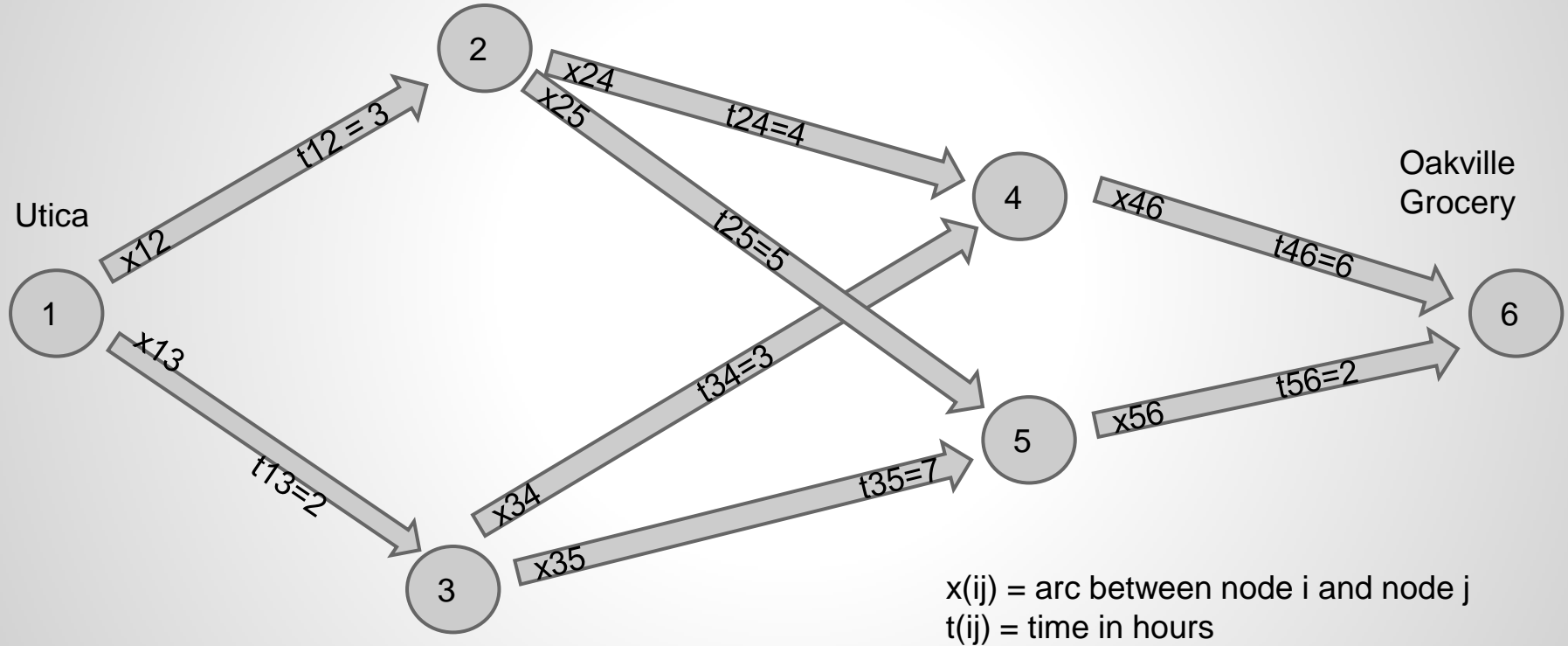
- Shortest Path Defined
- Word Problem
- Visual Model
- Equations
- Solution + Analysis

Definition of Shortest Path

- The shortest-path is finding the shortest path in a network from one node (or set of nodes) to another node (or set of nodes).
- Can minimize: **distance**, **cost**, and/or **time**.



Shortest Path Example



Shortest Path Example

Objective Function:

$$\text{Min } 3x_{12} + 2x_{13} + 4x_{24} + 5x_{25} + 3x_{34} + 7x_{35} + 6x_{46} + 2x_{56}$$

Shortest Path Example

Balance Equations for Nodes:

Origin:

$$x_{12} + x_{13} = 1$$

$$x_{23} + x_{25} - x_{16} = 0$$

$$x_{34} + x_{35} - x_{13} = 0$$

$$x_{34} + x_{35} - x_{24} - x_{34} = 0$$

$$x_{56} - x_{25} - x_{35} = 0$$

Destination:

$$x_{46} + x_{56} = 1$$

LINGO Format

Model:

!Shortest Path Problem;

!Objective Function;

Min = 3*x12+ 2*x13+ 4*x24+ 5*x25+ 3*x34+ 7*x35+ 6*x46+ 2*x56;

!Subject to;

x12+ x13= 1;

x24+ x25 - x12= 0 ;

x34+ x35 - x13= 0 ;

x34+ x35 - x24 - x34= 0;

x56- x25- x35= 0 ;

x46+ x56= 1;

End

LINGO Results

Global optimal solution found.

Objective value: 10.00000
Infeasibilities: 0.000000
Total solver iterations: 1

Model Class: LP

Total variables: 8
Nonlinear variables: 0
Integer variables: 0

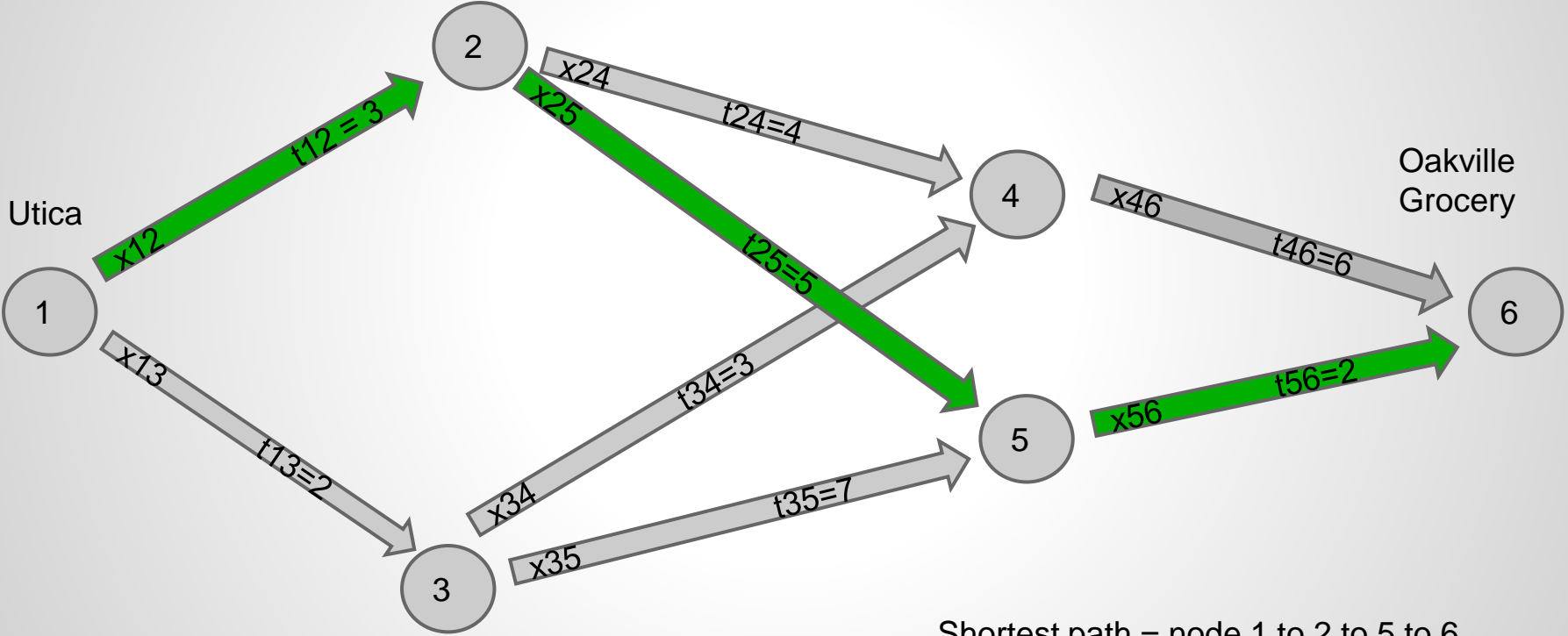
Total constraints: 7
Nonlinear constraints: 0

Total nonzeros: 23
Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
X12	1.000000	0.000000
X13	0.000000	0.000000
X24	0.000000	0.000000
X25	1.000000	0.000000
X34	0.000000	0.000000
X35	0.000000	3.000000
X46	0.000000	1.000000
X56	1.000000	0.000000

Row	Slack or Surplus	Dual Price
1	10.00000	-1.000000
2	0.000000	-5.000000
3	0.000000	-2.000000
4	0.000000	-3.000000
5	0.000000	2.000000
6	0.000000	3.000000
7	0.000000	-5.000000

The Shortest Path

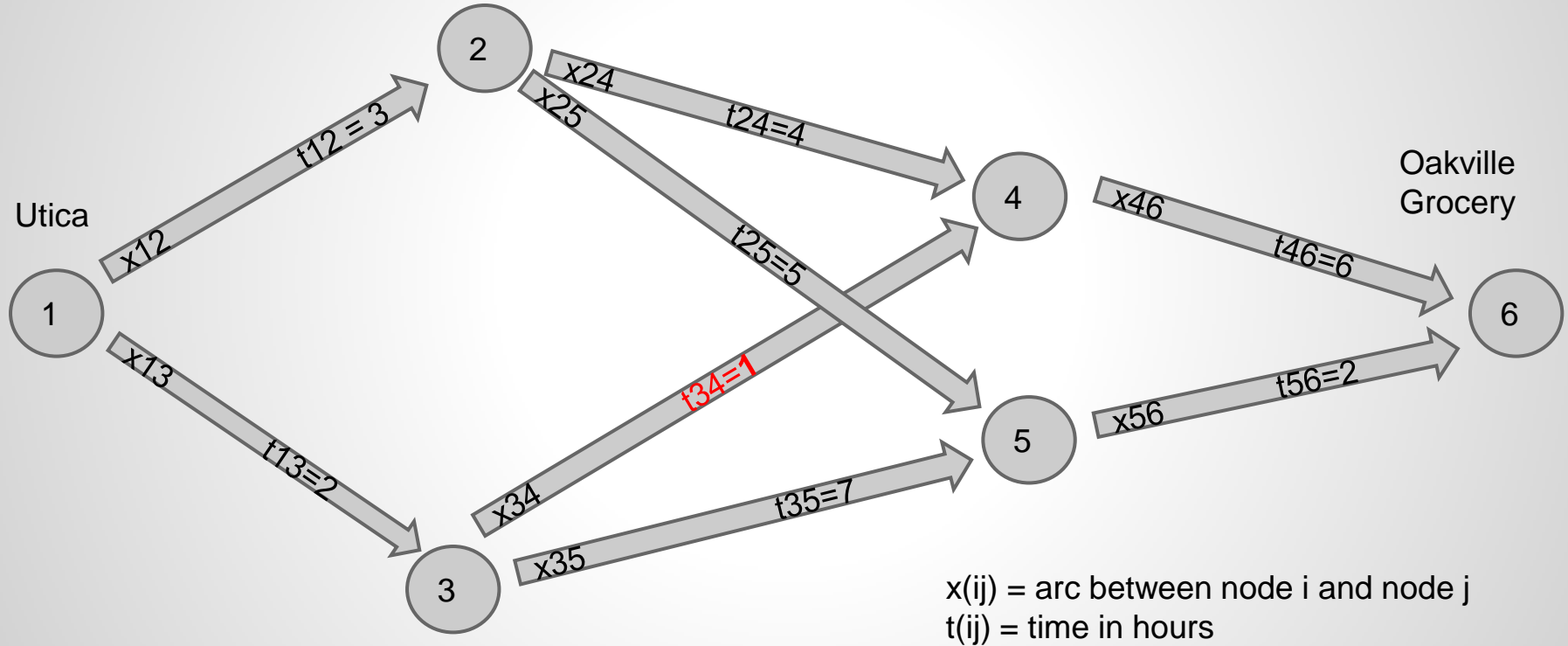


Shortest path = node 1 to 2 to 5 to 6
Time = 10 hours

BUT

What if we change
something?

Shortest Path Example



Now let's re-run the problem...

LINGO Format

Model:

!Shortest Path Problem;

!Objective Function;

Min = 3*x12+ 2*x13+ 4*x24+ 5*x25+ 1*x34+ 7*x35+ 6*x46+ 2*x56;

!Subject to;

x12+ x13= 1;

x24+ x25 - x12= 0 ;

x34+ x35 - x13= 0 ;

x34+ x35 - x24 - x34= 0;

x56- x25- x35= 0 ;

x46+ x56= 1;

End

LINGO Results

Global optimal solution found.

Objective value: 9.000000
Infeasibilities: 0.000000
Total solver iterations: 1

Model Class: LP

Total variables: 8
Nonlinear variables: 0
Integer variables: 0

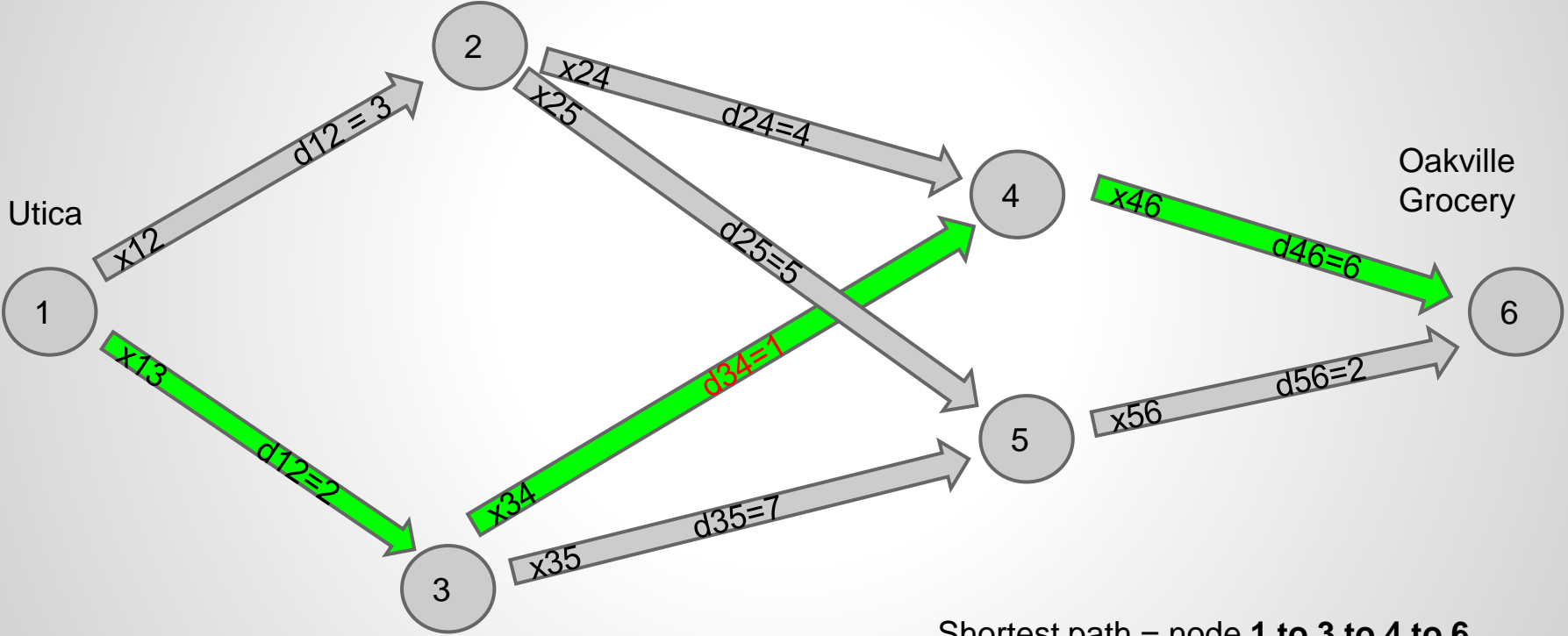
Total constraints: 7
Nonlinear constraints: 0

Total nonzeros: 23
Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
X12	0.000000	0.000000
X13	1.000000	0.000000
X24	0.000000	0.000000
X25	0.000000	1.000000
X34	1.000000	0.000000
X35	0.000000	6.000000
X46	1.000000	0.000000
X56	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	9.000000	-1.000000
2	0.000000	-3.000000
3	0.000000	0.000000
4	0.000000	-1.000000
5	0.000000	4.000000
6	0.000000	4.000000
7	0.000000	-6.000000

The Shortest Path



Shortest path = node **1 to 3 to 4 to 6**
Distance = **9** hours

THE END